Analyzing The Effects Of Fixed And Variable Costs

OVERVIEW

One of the most fundamental concepts in our daily lives is the notion of cost. We feel we know what it means to such a degree that we consider cost to be an objective trait of an activity or a physical object. That is, it seems like that, given a single set of facts, there should be no disagreement among reasonable people about what the cost of an item or activity was. However, this is not necessarily the case. Cost is just a concept. It is not an inherent characteristic.

One of the most fundamental questions in any organization is "what did it cost?" This can lead to potential problems because, although we can uniquely determine a stapler's color or its precise weight, if we manufacture it we cannot strictly determine its cost. At least we cannot do so with the same precision and veracity as we can with color and weight. Answering the apparently simple question "what did it cost?" often requires a rather lengthy and complex series of decisions and assumptions. To make matters worse for the manager, these decisions have typically been made by an accounting system designed for financial reporting, not for managerial decision making.

The purpose of this note is to explore some of the circumstances where decision-making requires managers to use costs differently from the way the cost accounting system provides them. Our goal here is to understand the implications of fixed and variable costs when making business decisions. Thus, the note will examine the analytical techniques involved in breakeven analysis and related profitability analyses. In order to reach that objective, however, we need to begin by examining how cost accounting systems define costs.

---

WHAT COST ACCOUNTING DOES

The primary job of a cost accounting system is to “collect” costs from throughout an organization and connect them to categories on the income statement and balance sheet. Most of the work of a cost accounting system is reasonably straightforward. It involves simple classification of costs into various balance sheet or income statement categories. Part of that process, for example, includes calculation of depreciation of non-current assets. The complicated part comes when the cost accounting system needs to determine the cost of a specific item in inventory (or sold from inventory) or the cost of a specific service job performed for a customer. For that reason, cost accounting systems are generally thought of as being related to production activity.

Cost accounting systems determine costs after the fact. That is, they collect the costs associated with, for example, all of the manufacturing activity during the year in a single factory. These costs are then matched in a variety of ways to the different goods that were produced as part of that activity. Some of these costs are easy to trace to specific goods. Others can never be matched precisely and are simply averaged in some way over the goods produced.

The easy-to-trace costs are called direct costs. By tradition, direct costs include direct materials (i.e., those materials that make up the final product) and direct labor (the cost of the labor time of the people who actually did the production, as opposed to supervisors, material handlers, inspectors, and so on). Occasionally, direct material cost plus direct labor cost is given the collective name prime cost.

Cost accounting systems usually categorize costs into three major cost components. Two of them are direct material cost and direct labor cost. The last major component of cost in a cost accounting system is overhead or burden. Overhead costs are all the costs in the factory other than direct material cost and direct labor cost. Overhead costs include, therefore, salaries of factory supervisors and managers, wages of support workers like data entry clerks and fork lift operators, depreciation on factory equipment, factory utility costs like heat and electricity, and even property taxes on the factory.
Technically, some overhead costs are direct costs. For example, although the electric bill does not directly reflect the cost of manufacturing one unit, if we placed a meter on each machine, we could record exactly how much of the consumed power was used by a single manufacturing step for a single item. However, this would never be done in practice because it would be too cumbersome and the cost of collecting the data would be too huge. In practice, all of these overhead costs are matched to products using some formula approach. This method is known as **cost allocation**.

Cost allocation is, to a great extent, the most troublesome aspect of cost accounting. The objective is to associate shared or **common costs** with production output through some quantifiable relationship. For example, a cost accounting system tries to address the question of how large a share of the costs incurred for building depreciation “go with” the production of each pair of jeans made in a clothing factory.

### COST TERMS AND DEFINITIONS

A few of the important terms used by cost accountants have already been introduced. These include prime costs, common costs, direct material costs, direct labor costs, and overhead or burden. A few other basic terms and their definitions are presented in this section.

**Product versus Period Costs**

Cost accounting systems are concerned with **product costs**. Under Generally Accepted Accounting Principles (GAAP), these costs include all and only the costs related to producing goods. In essence, they are all costs incurred in and for the factory. Product costs are shown on the balance sheet as inventories and are expensed on the income statement as cost of goods sold. **Period costs**, on the other hand, are never shown directly on the balance sheet. They are actually expenses incurred during the period. They are all non-factory costs that qualify as expenses. For example, their purchase of a new office computer would be considered an investment, not a period cost. The
depreciation on that computer, however, would be a period cost. Other period costs include marketing and promotion costs, research and development costs, sales commissions, general administration salaries, and so on.

The clarity of such classifications is sometimes less sharp than we would prefer it to be. For example, what is the appropriate classification of the wages of a clerk who records sales orders and initiates production against those sales orders? Strictly speaking, the first task is a period cost and the second task is a product cost. However, it is the unusual cost accounting system that will make such a detailed distinction.

**Fixed and Variable Costs**

All of the cost terms discussed thus far are critical to cost accounting. The distinction between fixed cost and variable cost, however, is not critical to cost accounting. In fact, cost accounting systems often ignore this pair of definitions. Perhaps this is because cost accounting systems take a long-term view of costs. As we know from economics, all costs are variable in the long term. In that case, the distinction does not exist. However, practically speaking, all decisions have some short-term aspect to them. All predictions of future costs have very real fixed and variable cost dimensions.

From the traditional definition, total fixed cost is fixed in relationship to variations in the volume of production and sales. That is, no matter how many units we manufacture and sell, total fixed cost will always be the same. Conversely, total variable cost rises and falls in direct proportion to increases and decreases in the volume of production and sales. No matter how many units we produce, total variable cost will be the number of units produced multiplied by a constant.

Let us examine an example. Assume our company manufactures something very simple, like the plastic outer shells of computer monitors. Let us say that the variable cost of each shell is $10 per unit. The total fixed cost for our injection molding machine is $500 of depreciation per month. If we make ten units this month, the total variable cost for the month is $100 ($10 x 10 units) and total
fixed cost for the month is $500. If we make 100 units this month, then total variable cost for the month is $1,000 ($10 x 100 units), but total fixed cost for the month is still $500. Produce 1,000 units in a month and total variable cost goes up to $10,000 ($10 x 1,000), but total fixed cost still remains at $500.

The distinction between fixed and variable costs would be an easy one, except for the fact that this definition only holds in relationship to production and/or sales volume. **Fixed costs can and do change**, but they do not change simply because production or sales volume changes. For example, one kind of fixed cost is annual depreciation of factory machinery. If a new machine is purchased, annual depreciation will increase. If an old machine becomes fully depreciated, annual depreciation will decrease. In both cases, fixed costs change from one year to the next. However, in neither case did the change occur because production or sales volume went up or down. Thus, the critical issue in determining what is a fixed cost and what is a variable cost is: does cost for that category vary in direct proportion to production and sales volume? Does the total cost go up if we make one more unit and go down if we make one less unit? If so, it is a variable cost. If not, it is a fixed cost. If the total cost changes in the right direction, but to a lesser extent, it is a **mixed cost**, made up of both fixed part and a variable part.

There is another dimension of the definitions of fixed and variable costs that is at odds with cost accounting and cost numbers produced by cost accounting systems. While cost accounting carefully separates product (factory) costs from period (office) costs, the general definition of fixed cost and variable cost includes both product and period costs. Some period costs are fixed each period, like management salaries. Some period costs vary with the level of sales, like order processing costs or sales commissions. The fixed and variable distinction is not limited to the factory.

Fixed and variable costs have an interesting kind of relationship to manufacturing activity. If a cost is fixed in relation to production volume, then average fixed cost per unit falls as volume increases. That is, if you buy a machine and use it to make one item before throwing it away, then the cost of the
unit you made includes the whole purchase price of the machine, say $1,000. If you make two units before throwing the machine away, each unit shares, on average, half of the purchase cost ($1,000/2 = $500). Obviously, the more units made with a single piece of equipment, the smaller each unit's share of that cost is. Thus, as production volume increases, average fixed cost per unit falls. On the other hand, since the variable cost of an activity is the amount by which total cost goes up as activity volume increases, the average variable cost per unit is constant. In summary, the following relationships hold:

<table>
<thead>
<tr>
<th>Total</th>
<th>per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed costs</td>
<td>constant</td>
</tr>
<tr>
<td></td>
<td>falls with increased volume</td>
</tr>
<tr>
<td>Variable costs</td>
<td>increases with volume constant</td>
</tr>
</tbody>
</table>

A good rule of thumb for considering fixed and variable costs is that each category has a “natural” form -- the one where it is constant. Thus, to best understand fixed costs, they should be examined as a total. Per unit fixed costs are “artifacts;” they are always the result of taking the natural fixed cost total and dividing it by the number of units produced. Whenever you encounter a fixed cost expressed in “per unit” terms, convert it to its natural total form. To best understand variable costs, they should be examined on a per unit basis. Total variable costs are always determined by taking the natural variable cost per unit and multiplying it by the number of units produced. Thus, when you encounter variable cost as a total, you should convert it into the unit cost.

Let's look at an example. You work for a company that makes plastic components for the telecommunications industry. One operation is a dedicated plant that makes only the plastic bodies for a single line of cell phones. Your boss has asked you to give her projected costs of producing 1,000,000 units of that plastic body next year. She also wants you to identify what the average cost per unit will be, broken down into fixed cost per unit and variable cost per unit. She wants the information within the hour. You immediately obtain the following
information from the cost accountant; more details will not be available until tomorrow.

1. 800,000 units were produced last year.
2. Total production costs were $700,000 last year.
3. Average fixed costs per unit were $0.25 last year.
4. All input cost rates are expected to remain the same, including total fixed cost.

Do you have enough data to do the projections? Yes, but first you must put the costs into their "natural" forms. You are given that fixed costs per unit averaged $0.25 last year. Average fixed costs per unit are the result of dividing total fixed cost by the total production volume. Thus, if we multiply the fixed cost per unit by the total production volume, we can reverse the process and find total fixed cost. This means total fixed cost last year was 800,000 units x $0.25, or $200,000.

Now you need to find variable cost per unit. We know total costs were $700,000. Of that total, $200,000 was fixed. Therefore, $500,000 must have been the total for variable cost. This total was simply the result of multiplying the variable cost per unit by the number of units made. Once again, we reverse the process to find the variable cost per unit. This means variable cost per unit last year was $500,000/800,000 units = $0.625 per unit.

Since the cost rates are not expected to change, these numbers should apply to next year, too. Therefore, total fixed cost should be $200,000 and the new total variable cost should be 1,000,000 units x $0.625 per unit, or $625,000. You can tell your boss that production of 1,000,000 units should have a total cost of about $825,000. At that total, the average cost per unit should be $825,000/1,000,000 units = $0.825 per unit. Furthermore, variable cost per unit will remain at $0.625, since no change in rates is expected. However, the fixed cost per unit will change. Since the same total is being used to cover a higher volume of production, fixed cost per unit will fall. The new average fixed cost per unit will be $200,000/1,000,000 units, or $0.20 per unit. This could also be
determined by subtracting the variable cost per unit from the average total cost per unit: $0.825 - $0.625 = $0.20.

Once again, note that variable cost per unit and total fixed cost do not change as the production volume moves to a higher level. However, both total variable cost and fixed cost per unit do change. Since the new volume is higher, so is total variable cost. Since the same total fixed cost is shared by more units of output, fixed cost per unit goes down.

Analyzing Fixed and Variable Costs

It is obvious that the strict definitions of fixed and variable costs cannot apply to any real business. At some point, the production capacity made possible by the fixed costs will be reached. Once that point is reached, fixed costs will have to be increased in order to allow a greater volume of production. That is, if a factory's capacity is 1000 units per day, the cost to make 1100 units per day will necessarily include the rental of more factory space, the hiring of an additional supervisor, or the depreciation of additional machinery. Thus, fixed costs will move to a higher plateau. This is illustrated in Figure 1. Likewise, at very low production volumes we may move to higher costs for materials since they are purchased in small lots. At very high production levels, we may have to hire more workers and change the average cost of labor per hour (and, due to inexperience, the labor hours per unit). This is illustrated in Figure 2.

The situation can be even more complicated. Some costs that "look like" fixed costs in a close-up, begin to look like variable costs from afar. Think of the cost of manufacturing supervisors' salaries. If supervisors are only allowed to oversee four workers, then as production volume moves up and more workers get added, total supervisor salaries must increase, too. Given that each worker can make 100 cell phone bodies a day, then supervisors' salaries is a fixed cost if the production range happens to be 100 to 400 units per day. At four hundred
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

and ten units per day, a fifth worker would have been added. This would require a second supervisor. Thus, in the range 100 to 800 units per day, the cost category of supervisors' salaries is fixed at two levels. If the range is 100 to 1600 units per day, supervisors' salaries move among four plateaus, as in Figure 3. Each of these plateaus is called a step, and the costs are called step-variable costs for obvious reasons.

While economists tend to draw production cost curves with curved lines, accountants usually employ straight lines. Accountants have been aware of the shortcomings of such an approach for many years, but they have felt comfortable using the fixed-variable distinction because they know that they usually do not
need to worry about the extremes of the range. In fact, they are usually looking at only a small portion of the possible range of levels of production. We call this set of volumes under study the **relevant range**. If we combine the data in Figures 1 and 2, we get the total cost graph in Figure 4. Furthermore, if we limit our focus to the relevant range of 500 to 1000 units per day, we see a relatively straight line segment within that range.

![Figure 1-4: Total Production Costs](image)

There are a few things we should observe about Figure 1-4 before we leave it. First, note that we can determine a formula that will reproduce the graphed line we see within the relevant range. This formula is called, simply, the total cost function or total cost formula. We have already applied this formula to our computation of expected costs for cell phone bodies. It is expressed formally as:

\[
\text{Total Cost} = \text{Total Fixed Cost} + (\text{Variable Cost per Unit} \times \text{Unit Volume})
\]

It is a bit difficult to determine from the small graph picture above, but the formula in the relevant range of Figure 4 is:

\[
\text{Total Cost} = $1,000 + ($2.00 \times \text{Unit Volume})
\]

This cost formula indicates that, within the relevant range, total fixed cost is $1,000 and variable cost per unit is $2.00. Thus, at 500 units, total expected cost is $1,000 + ($2.00 \times 500) = $2,000. At 1,000 units, the total expected cost would be $1,000 + ($2.00 \times 1,000) = $3,000. Once we have determined the "natural"
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

fixed and variable cost numbers, we can draw a predictive cost graph which employs only the cost formula for the relevant range. Such a graph is shown in Figure 5.

![Figure 1-5](image)

The data for total fixed cost and for variable cost per unit could be derived in a variety of ways. Typically, a company would use historical data or, for a new facility or product, engineering data. In order to determine these numbers, it is important that we pay close attention to the definitions of fixed cost and variable cost. This is most critical when an "account method" is used. Under the account method, someone examines each of the various company expenditure accounts and classifies them as either fixed or variable. Since many accounts include mixed costs (i.e., costs that are partially fixed and partially variable), this method is fairly crude. If enough data is available, the most effective way of determining the critical cost numbers is by using simple regression analysis. The inputs are total period cost for the dependent variable and total period production/sales volume for the independent variable. The "intercept" from the analysis is total fixed cost. The "slope" is variable cost per unit.

No single category of expenditure is likely to be a "pure" fixed cost or a "pure" variable cost. However, if a set of assumptions is satisfied for a situation, we can characterize costs in terms of fixed and variable and use the distinction to create a cost formula like the one above. We can then make some projections about costs and profits. Here is the set of assumptions:
1. First, we must be able to assume that costs are (nearly) "linear" within the relevant range. If the curves that map costs to manufacturing activity are curly, then fixed-variable analysis will not apply. The graphs must be characterized by straight lines.

2. Second, we must assume that all of the activity levels to be considered are within the relevant range. If they fall outside the relevant range, the fixed-variable relationships are no longer in effect.

3. Third, we have to assume that we sell everything we make. That is, production equals sales.

4. Finally, we must assume that the job we did of splitting total costs into a fixed component and a variable component does reflect the actual situation. If the relationships are misleading, so is the analysis.

If we feel comfortable that the situation we are examining meets these requirements, we can use the total cost formula to do some very useful analyses.

The Basic Analytical Model using Fixed and Variable Costs

Now that we have simplified the production situation into straight line graphs of costs where we sell all we make, we can turn a simple formula for profit

\[
\text{Pre-tax Profit} = \text{Total Revenues} - \text{Total Cost}
\]

into a rather powerful tool. By substituting into the formula, three definitions --

\[
\text{Total Revenues (TR)} = \text{Selling Price per unit (SPu)} \times \text{Units sold}
\]

\[
\text{Total Cost (TC)} = \text{Total Fixed Cost (TFC)} + \text{Total Variable Cost}
\]

\[
\text{Total Variable Cost (TVC)} = \text{Variable Cost per Unit (VCu)} \times \text{Units sold}
\]

-- it becomes:

\[
\text{Pre-tax Profit} = (\text{SPu} \times \text{Units sold}) - (\text{VCu} \times \text{Units sold}) - \text{TFC}.
\]

Note, by the way, that total fixed cost includes both product costs and period costs, as does total variable cost.

The equation above defines a kind of income statement. That is, the first line is total revenue (SPu * Units sold) from which total variable cost (VCu * Units sold) is subtracted. Then total fixed cost is subtracted. In an income statement based on this model, there is no gross margin, since costs are split into the two categories of fixed and variable costs, not product and period costs. There is, however, an analogous step in the statement, contribution margin.
Contribution margin is the difference between total revenues and total variable costs. A contribution format income statement looks like this:

\[
\begin{align*}
\text{Total sales revenues} & \quad \text{less} \quad \text{Total variable costs} \\
\text{equals} & \quad \text{Total contribution margin} \\
\text{less} & \quad \text{Total fixed costs} \\
\text{equals} & \quad \text{Net Income before taxes}
\end{align*}
\]

Since, by earlier definition, Total Revenues and Total Variable Costs are both functions of units sold (i.e., they are variable), total contribution margin is also a variable factor. As such it has a unit-level version, contribution margin per unit (CMu). CMu is total contribution margin divided by the production/sales volume. More directly, it is the difference between the unit selling price and the variable cost per unit:

\[\text{CMu} = \text{SPu} - \text{VCu}\]

This puts the final twist on our simplified profit formula:

\[\text{Pre-tax Profit} = (\text{CMu} \times \text{Units sold}) - \text{TFC}.\]

Now we can use this formula to project profits at different levels of activity (assuming we stay in the relevant range).

**Breakeven Analysis**

The first application of the formula is the calculation of units sold to break-even. Break even is when neither a gain nor a loss exists. It is the point when total revenues equals total cost (or where total contribution margin equals total fixed cost).

In terms of the graphical analysis we used in the previous section of this note, a breakeven analysis adds another dimension to Figure 1-5, the revenue line. The result, shown in Figure 1-6, is often called a cost-volume-profit graph. The breakeven point is the point where the total cost line and the total revenue
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

line cross. At higher production/sales volumes, there is a profit. At lower volumes, there is a loss.

We can use the profit formula to calculate the production/sales volume point where break even is achieved. To find the break even point in units produced and sold, we solve for units sold in the formula, after substituting zero for profit:

\[ 0 = (CMu \times \text{Units sold}) - TFC \]

\[ CMu \times \text{Units sold} = TFC \]

\[ \text{Units sold} = \frac{TFC}{CMu} \]

In the example we have been using for the figures in this note, we know that the total cost formula is \( TC = 1,000 + (2.00 \times \text{Unit volume}) \). If we also know that the unit selling price is \$4 per unit, we can determine the exact break even point on the graph in Figure 1-6.

First, we need to determine \( TFC \). This is available directly from the total cost formula. It is the "intercept" of \$1,000. Next, we need to determine \( CMu \). By definition, \( CMu = SPu - VCu \), so \( CMu \) in this case is \$4 - \$2 = \$2 \). Now we can substitute the values into the formula:

\[ \text{Units sold} = \frac{1,000}{2} = 500 \text{ units} \]

Thus, the break even point occurs where 500 units are made and sold. At that point, sales revenue is \( 500 \times \$4 = \$2,000 \). Total cost is \$1,000 plus \$2 \times 500, or a total of \$2,000, too.
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

If we want to figure the break even point in sales dollars, we can simply multiply the units sold times the selling price as we did above to find $2,000, or we can use the contribution margin ratio (CMr) in place of the CMu. The CMr is the ratio CMu/SPu or TCM/TR. It is the percentage of each sales dollar that is left over after paying for variable costs. Applying this approach in the example in Figure 1-6, we get the following:

\[
\text{Revenue at break even} = \frac{\text{TFC}}{\text{CMr}}
\]

In the example, CMu is $2 and SPu is $4, so CMr = 2/4 = .50 (or 50%). Therefore, revenue at break even is $1,000/.5 = $2,000.

Break Even and Operating Leverage

Determination of a company's, factory's, or project's break even point is an important part of profit planning. Break even analysis is often the starting point for examining the soundness of a project. If the break even point is higher than the likely sales volume, then the project is not acceptable under its proposed cost structure.

Break even analysis can be nicely incorporated into an examination of alternative ways of producing a product or providing a service. Organizations typically have a variety of choices as to how to make or do something. Traditionally, there is a trade-off between labor-intensity and capital-intensity to be considered. In the terms we are using here, this is a trade-off between a high variable cost/low fixed cost approach and a low variable cost/high fixed cost approach.

Consider the manufacture of a simple printed T-shirt which would sell for $10 each. The T-shirts could be made by a skilled worker using a simple silk screen frame. Suppose the worker could make ten shirts per hour and was paid $12 per hour. You keep the worker only until the number of shirts needed for the day is reached. Thus, you pay the worker about $1.20 per shirt ($12 per hour / 10 shirts per hour). The white T-shirts (the raw materials) cost $7 each and the printing might use $0.25 of ink. The total variable cost for each finished T-shirt would be $1.20 for labor + $7 for the blank T-shirt plus $0.25 for ink = $8.45.
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

These T-shirts would then have a CMu of $1.55 each ($10 selling price - $8.45 each variable cost per T-shirt). If the frame cost $31, then the break even point would be 20 T-shirts made and sold ($31 TFC/$1.55 CMu =20). Suppose, as an alternative, the manufacturer purchased a fully-automated T-shirt printer for $2,500. Since it is fully automated, it can produce 20 T-shirts per hour using less skilled labor. Perhaps the less-skilled labor earns $10 per hour. In this case, total variable cost per T-shirt is $0.50 for labor plus $7.00 for the blank T-shirt plus $0.25 for ink = $7.75. Thus, the CMu for the automated approach is $2.50. In the automated case, the breakeven point is 1,000 T-shirts ($2,500 TFC/$2.25 CMu ), much higher than the skilled labor case. But what if there is demand for 10,000 of these T-shirts at this price? If so, the labor-intense approach earns a TCM of $15,500 ($1.55 x 10,000 units) and a profit of $15,469 ($15,500 - $31). However, the capital-intense approach earns $25,000 in TCM ($2.50 x 10,000 units) and a profit of $22,500 ($25,000 - $2,500).

The ability of a high fixed cost/low variable cost manufacturing approach to earn large profits at volumes well above break even is due to its bigger CMu. The ability of such an arrangement to generate faster increases in profits (or faster decreases in profits, as volume falls) is called operating leverage. Like financial leverage, high operational leverage is neither good nor bad on its own. Here goodness is judged in the distance between the break even point and the demand volume.

The success of mass production in the early part of the 20th century can be attributed to the positive effects of operational leverage. Until Ford, automobiles were manufactured by highly skilled artisans using very simple tools. They would cut and shape each fender by hand. This made production output slow and variable cost of production very high. Fixed costs were minimal. Nonetheless, prices had to be extremely high to allow a positive contribution margin. Ford's mass production system utilized a large fixed cost base: detailed planning, support and supervisory staff, plant management, large plants, and great automated machines. In Ford's plants, a special machine cut and stamped out a dozen identical fenders all at once. Due to this systematization and
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

Automation, however, Ford could employ unskilled workers as labor. Thus, Ford could maintain strong contribution margins while lowering prices as volume grew.

**Target Profit /Volume**

The break even point for a company, a product line, or a project is an important milestone, but most organizations are interested in something more -- an acceptable level of profit. The analytical methods we have been exploring here can be applied to examining potential profit, as well. We can extend the logic of break even analysis to calculate the units that must be sold (or the revenues earned) in order to yield a certain profit.

Earlier, we showed how to determine profit in CM terms:

\[
\text{Pre-tax Profit} = (\text{CMu} \times \text{Units sold}) - \text{TFC}
\]

We can algebraically re-arrange the pieces of this formula to read:

\[
\text{TFC} + \text{Pre-tax Profit} = \text{CMu} \times \text{Units sold}
\]

After dividing each side of the equation by CMu, we find that we can calculate the number of units to sell to achieve a given level of pre-tax profit by the formula:

\[
\text{Units sold} = (\text{TFC} + \text{Target Pretax Profit})/\text{CMu}.
\]

This formula can be adjusted for taxes, too:

\[
\text{Units sold} = \frac{\text{TFC} + \left(\frac{\text{After Tax Profit}}{1-\text{tax rate}}\right)}{\text{CMu}}
\]

There are many other variations. As you can see, this is a very flexible estimating tool, as long as the assumptions are pretty close to reality. We have one more set of extensions of this cost-volume-profit analysis model to examine in the note. Before we do so, however, we will review a summary example of these break-even and target profit formulae.

Such-a-Snack produces a wide range of snack chips. Bob, the company’s owner, has asked for your help to see why he isn’t making as much money as he’d like. He presents you with the following information for ruffled potato chips, one of the company’s big sellers:

Such-a-Snack made and sold 600,000 bags of ruffled potato chips last year. At that level of production and sales, the following cost data was obtained:

**Manufacturing costs:**
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

Selling price per 12 oz bag: $1.75

Variable costs/bag:
- Potatoes $0.65
- Vegetable oil $0.10
- Salt, etc. $0.05
- Packaging $0.05

Fixed manufacturing costs/bag $0.50

Office costs:
- Variable Selling costs last year $90,000
- Fixed selling & administrative/year $150,000

Bob wants to know how many bags of ruffled potato chips must be sold for Such-a-Snack to break even on that product line. He also wants to know how many bags need to be sold to generate a profit of $100,000, but he has not told you if he means before-tax or after tax. You had better do both calculations, just to be safe. Assume a tax rate of 40%. Finally, Bob wants to know what the profits will be if, as currently predicted, sales and production volume increases by 10% this year to 660,000 bags. How many bags would this be above breakeven (this number is called the margin of safety)?

In order to address Bob’s concerns, you need first to assemble all of the appropriate data. This means you must find CM per bag and TFC. In order to find CM per bag, you need SP per bag, which is given, and VC per bag, which you have to determine. We’ll do each of these things before we start plugging numbers into the formulae.

We know that “office” fixed costs total $150,000. Manufacturing fixed costs are given as an average for the 600,000 bags made last year. Therefore, we need to multiply the $0.50 per bag average by the 600,000 bags to determine that total fixed manufacturing cost was $300,000. Thus, total fixed cost is $300,000 + $150,000 = $450,000.

On the manufacturing side, we see that variable costs add up to $0.85 per bag (0.65 + 0.10+ 0.05 +0.05). There are variable selling costs, too. For the 600,000 bags sold, they totaled $90,000 last year, so the per bag amount is $90,000/600,000 bags = $0.15 per bag. This brings the total for VCu to $1.00
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

per bag. The selling price is $1.75 per bag, so the CMu is 1.75 - 1.00 = $0.75 per bag.

We can now determine that the break even point is:

$$\text{TFC/CMu} = \frac{450,000}{0.75 \text{ per bag}} = 600,000 \text{ bags.}$$

This is the same as the sales volume, so such-a-Snack is making NO profit on this product at the moment!

Making a profit of $100,000 before tax means Such-a-Snack has to earn another $100,000 of CM above break even. If we put that into the equation above, the result is:

$$\frac{(\text{Pre-tax Profit + TFC})}{\text{CMu}} = \frac{(100,000 + 450,000)}{0.75} = 733,333.33 \text{ bags}$$

Technically, they cannot sell .33 bags, so the sales point is 733,334 bags. That is more than a 20% increase over last year. To earn an after-tax $100,000 will be even more difficult. To get $100,000 after a tax bite of 40%, you have to start with $100,000/(1-.40) or $100,000/.60 = $166,666.66 before tax. If we put that value in the formula, we get:

$$\frac{(450,000 + 166,667)}{0.75 \text{ per bag}} = 822,223 \text{ bags}$$

Things look pretty bleak for Such-a-Snack if the company is looking for $100,000 after-tax from this product line. Let's look at what the more prudent profit expectation is. We know 660,000 bags represents a margin of safety of 60,000 bags, or 10%. We can calculate pre-tax profit directly using the CMu. Since each bag sold earns a CMU of $0.75, 660,00 bags would gather a total CM of $495,000. This figure already accounts for the total variable costs, so only total fixed cost needs to be subtracted. Thus, expected pre-tax profit at 660,000 bags would be $495,000 TCM - $450,000 TFC = $45,000. Bob probably won't want to know the after tax net, but it would be 60% of the $45,000, or $27,000. If this is one of Such-a-Snack's best lines, they better find out how to increase demand or how to cut costs!
The Extended Model -- Differential Analysis

Differential analysis is a name given to a wide variety of cost-based comparisons of alternatives. Traditionally, differential analysis has been classified into five major groups: special order decisions, make/buy decisions, drop/add decisions, sell-or-process-further decisions and resource allocation decisions. The premise underlying all differential analysis is that selecting the best alternative depends only on the costs and revenues that differ across alternatives. In almost every decision in the five classes above, differences will exist in total contribution margin. However, in many cases there will also be differences in fixed costs. Opportunity costs may affect both contribution margin and fixed costs.

Our objective here is not to develop analytical methods for each of these classes of decisions. However, these analytical techniques may prove to be useful as part of the course project, so three of them are outlined below. In any event, you will not be directly tested on the techniques described below.

Special Order Decisions

Although the Robinson-Patman Act prohibits price discrimination within a market, pricing differences for special cases do arise. Different markets, competitive bids, and "private label" manufacturing all may result in selling nearly identical products at differing prices. Special orders, that is, orders for limited numbers of goods at what are often deeply discounted prices, are often quite attractive to sellers, especially when producing for the order allows the seller to fill-in idle manufacturing capacity.

The numerical analysis often boils down to a simple idea: any increase in contribution margin shows up on the bottom line, since fixed costs are a constant. As long as the special order does not cause an increase in fixed costs, any selling price above the variable costs per unit will increase profit!

The hang-ups in this decision come from lack of capacity. What if your plant has enough idle capacity to produce 2000 extra units beyond normal sales...
and a special order bid comes in for 1,500 units at $75 each? If the current normal sales price is $90 each and the unit variable cost is $65, would you make money or lose on the deal?

Traditional cost account reports on the product would not be very helpful for this decision. They would tell you the unit price ($90), but they would tell you the average cost per unit, not the variable cost. Thus, if the average fixed cost per unit was $11, the cost accounting report would indicate that the product cost to company $76 per unit to make. The deal would be rejected. However, the deal calls for no more fixed costs than before. Total fixed costs will not go up, so the only important data is the $75 in revenue per unit and the $65 in variable costs per unit. In fact, taking the deal will increase pre-tax profit by $15,000 (1,500 units multiplied by a contribution margin per unit of $10).

If idle capacity was only 1,000 units, what would happen then? The company would have to sacrifice 500 units of normal sales if it took the special bid. Thus, it would lose $12,500 of CM from normal sales: 500 units at a CMu of $25 each ($90-$65). However, the firm would gain $15,000 in contribution margin for the special order. This profit analysis shows a net increase in pre-tax profit of $2,500, but there may be many problems associated with that course of action if customer satisfaction, loyal distribution systems, or full service is your goal.

**Make/Buy Decisions**

Many manufacturers are regularly faced with decisions as to whether to make a component or buy it. In fact, some manufacturers do not even make any part of the entire product, believing that their special competence comes in designing or marketing the product, not building it.

Once again, the idea is to see what increment to total cost occurs when the product is made internally and what increment occurs when it is made externally. The latter cost is often easy to determine if you have a ready supplier. You ask them for a bid. The internal savings is usually a bit more difficult to determine. It is important to remember, however, that the cost system has
produced costs that share all overhead, including all fixed manufacturing costs. The savings that will occur from not making the part consists of the product's variable costs, any fixed costs that can be avoided (e.g., layoffs of supervisors), or any new opportunities for income that come from freeing resources (e.g., rent out floor space).

**Drop/Add Decisions**

Drop/Add decisions focus on whether a business segment (a product line, a sales office) adds income to the organization or whether it is a net drain. The issue is what income is lost if the segment goes away or what income is added if the segment is made part of the company.

Since business segments generally have some dedicated fixed costs, there will not only be variable costs to analyze in this situation. We also have to separate out the "direct" fixed costs of the segment from those that are shared with other parts of the organization. For example, if a segment to be dropped is a product line, then all fixed costs of *sales offices* will stay. If the segment is a sales office, then all of the costs of the manufacturing plants will stay (unless one plant sells only through that office).

There is a standard detail report that provides this information. It is called a **segment margin income statement**. An example appears in Exhibit 1-1. Note that it is obvious from these two revisions of the same data that both product lines and both sales offices make positive contributions. Common fixed costs are not allocated to the segments. The bottom line in each segment column is its **segment margin**. If these numbers had included allocated amounts of common fixed costs, the issue might not have been so clear.

---

**Exhibit 1-1: SEGMENT MARGIN INCOME STATEMENT FORMAT**

<table>
<thead>
<tr>
<th>--BY PRODUCT LINE</th>
<th>TOTAL</th>
<th>ALPHA</th>
<th>BETA</th>
</tr>
</thead>
<tbody>
<tr>
<td>NET SALES</td>
<td>$830,000</td>
<td>$450,000</td>
<td>$380,000</td>
</tr>
<tr>
<td>- VARIABLE COST OF SALES</td>
<td>(400,000)</td>
<td>(230,000)</td>
<td>(170,000)</td>
</tr>
<tr>
<td>=CONTRIBUTION MARGIN</td>
<td>$430,000</td>
<td>$220,000</td>
<td>$210,000</td>
</tr>
<tr>
<td>-SEPARABLE FIXED COSTS</td>
<td>(180,000)</td>
<td>(100,000)</td>
<td>(80,000)</td>
</tr>
</tbody>
</table>
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

\[
\begin{array}{ccc}
\text{PRODUCT (SEGMENT) MARGIN} & \$250,000 & \$120,000 & \$130,000 \\
\text{- COMMON FIXED COSTS} & -(125,000) & & \\
\text{=NET INCOME} & \$125,000 & & \\
\end{array}
\]

**--BY OPERATING SEGMENT (SALES OFFICE IN THIS EXAMPLE)--**

<table>
<thead>
<tr>
<th></th>
<th>TOTAL</th>
<th>EAST</th>
<th>WEST</th>
</tr>
</thead>
<tbody>
<tr>
<td>NET SALES</td>
<td>$830,000</td>
<td>$430,000</td>
<td>$400,000</td>
</tr>
<tr>
<td>- VCOS</td>
<td>(400,000)</td>
<td>(200,000)</td>
<td>(200,000)</td>
</tr>
<tr>
<td>= CONTRIBUTION MARGIN</td>
<td>$430,000</td>
<td>$230,000</td>
<td>$200,000</td>
</tr>
<tr>
<td>- SEPARABLE FIXED COSTS</td>
<td>(100,000)</td>
<td>(55,000)</td>
<td>(45,000)</td>
</tr>
<tr>
<td>= SEGMENT MARGIN</td>
<td>$330,000</td>
<td>$175,000</td>
<td>$155,000</td>
</tr>
<tr>
<td>- COMMON FIXED COSTS</td>
<td>(205,000)</td>
<td>   </td>
<td></td>
</tr>
<tr>
<td>= NET INCOME</td>
<td>$125,000</td>
<td>   </td>
<td></td>
</tr>
</tbody>
</table>

**The Cost Management Context**

One of the major criticisms of the approaches described above is that they make too much sense. If managers get the habit of using these methods to solve their problems, they do not seek longer-term improvements. The cost analyses that these methods use are predicated on the idea that what calculates as lowest cost or highest profit is actually in the company's long-term interest. Unfortunately, this is far from the dominant case. Very often strategic issues or external constraints nullify some of the options which the analysis would promote. Good managers use these analytical tools not to tell them what to do, but to highlight problems or needs for improvement in the implementation of the options that they *must* pursue.

Cost management means spending to create income, not simply saving by keeping costs low. It also means managing costs at the global, firm-wide level, not just within a department. In some cases, managing costs does mean avoiding costs, but utilization of the traditional methods often leads to cost evasion. That is, managers try to find ways of not being assigned costs by the cost accounting system rather than trying to find ways of helping to eliminate the costs in the first place.

It is important that the analytical techniques discussed in this note be used with caution. Remember that the company's strategy, long-run developmental
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

needs, and other contextual characteristics often play a much more important role in the decision as to what action to take.

It should be fairly obvious at this point that cost accounting systems have a practical job to do under less than perfect circumstances. Financial reporting requires quantification of cost of goods sold and of inventories. These figures must reflect reasonable matching of all the costs of production to goods produced. In some cases the practical requirements force the use of methods or estimates that are more expedient than revealing of underlying "truth." However, that is the job of a cost accounting system.

Cost management is the task of making costs something they are not. That is, managing costs means changing them in such a way that the company makes more profit. Sometimes this means reducing costs, sometimes it means increasing costs. Cost management focuses on actions to be taken and strategic objectives to be pursued.
Solved Practice Problems

Problem 1
Pete’s Products makes plastic boxes for storing computer diskettes. Based on past experience, Pete has found that his costs are the following:

Materials per box made:
- White plastic 10 oz. @ $0.03/oz
- Clear plastic 3 oz. @ $0.05/oz
- Decals 1 set @ $0.10 per set

Labor and Machine Time:
- Labor $15.00/hour Each box takes 3 minutes to assemble
- Machine Time $30.00/hour Each box takes 1 minute of machine time

Other Costs:
- Rent $1,000 per month
- Supervision $2,500 per month
- Insurance $ 300 per month
- Utilities $ 600 per month
- Supplies $ 300 per month
- Maintenance $1,300 per month

Required:

1. What is the average variable cost per unit made?
2. What is the total fixed costs for the business?
3. Assume the following range of output volume in a month: 1,000 boxes; 10,000 boxes; 50,000 boxes; 100,000 boxes.
   Determine:
   a. Total variable cost for each volume of output.
   b. Total fixed cost for each volume of output.
   c. Total cost for each volume of output.
   d. Average variable cost per box made for each volume of output.
   e. Average fixed cost per box made for each volume of output.
   f. Average cost per box made for each volume of output.
Problem 2
The cost formula for producing clown noses is:

\[ TC = 0.15 \times V_m + 8,000 \]

where \( V_m \) is the number of units we make.

Required:

1. Describe what each piece of the formula means.
2. Determine the total cost if we make 1,000 noses; 5,000 noses; 8,000 noses; 16,000 noses; 25,000 noses.
3. Based on your answer to #2, what is the average cost per unit made for each of these output volumes?

Problem 3
Adams Corporation makes packets of checks used by bank customers to withdraw funds from their checking accounts. During the coming month, Adams Corp. expects to make 50,000 individual orders of checks. The expected costs for filling these orders are:

<table>
<thead>
<tr>
<th>Cost Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct materials (paper, ink)</td>
<td>$75,000</td>
</tr>
<tr>
<td>Direct labor (press operators)</td>
<td>$10,000</td>
</tr>
<tr>
<td>Overhead</td>
<td></td>
</tr>
<tr>
<td>Supplies</td>
<td>$5,000</td>
</tr>
<tr>
<td>Depreciation</td>
<td>$3,600</td>
</tr>
<tr>
<td>Indirect labor</td>
<td>$2,400</td>
</tr>
<tr>
<td>Power</td>
<td>$500</td>
</tr>
<tr>
<td>Supervision</td>
<td>$2,300</td>
</tr>
<tr>
<td>Insurance/fees</td>
<td>$700</td>
</tr>
<tr>
<td>Rent</td>
<td>$1,500</td>
</tr>
</tbody>
</table>

Assume that these costs are either strictly fixed or strictly variable.

Required:

1. Prepare a cost formula for the following costs:
   a. Direct materials
   b. Direct labor
   c. Variable overhead
   d. Total variable costs
   e. Total fixed costs
   f. Total costs
   g. Average total cost
2. For a planned level of production of 75,000 orders, calculate
   a. Total cost of direct materials
   b. Total cost of direct labor
   c. Total variable overhead
   d. Total variable costs
   e. Total fixed overhead
   f. Total cost
3. For a planned level of production of 75,000 orders, calculate
   a. Average cost of direct materials.
   b. Average cost of direct labor.
   c. Average variable overhead.
   d. Average variable cost.
   e. Average fixed cost.
   f. Average total cost.
4. Repeat numbers 2 and 3 if total output is planned to be 100,000 orders. What happens to the total costs? ...to the average cost per order produced? Why do these change (if they do)?

Problem 4.
Basic Products produces a single product. Last year's contribution format income statement shows the following:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales (45,000 units)</td>
<td>$1,710,000</td>
</tr>
<tr>
<td>Less: Variable costs</td>
<td>684,000</td>
</tr>
<tr>
<td>Contribution Margin</td>
<td>1,026,000</td>
</tr>
<tr>
<td>Less: Fixed Costs</td>
<td>525,000</td>
</tr>
<tr>
<td>Net Income (before tax)</td>
<td>$ 501,000</td>
</tr>
</tbody>
</table>

Required:
1. What is the variable cost per unit? The selling price per unit?
2. Compute the breakeven point in units and sales dollars.
3. Suppose that Basic Products is considering buying a new robotics manufacturing line. The line will increase fixed costs by $200,000 per year, but will lower variable costs to 30% of revenues. What will be the new breakeven point in units and sales dollars assuming that the investment is made?

Problem 5.
Frank's Nursery is thinking about launching a new product line, hothouse vegetables. These vegetables would be grown indoors, and would be available all year round. Frank figures he could "turn" his inventory of plants about three times a year, on average, so he could make a lot of money if the cost of setting up the greenhouse and heating it isn't too large.

Frank has asked you to help him figure out just how many crates of tomatoes and related vegetables he has to sell to cover his investment and make some money besides. The facts he gives you are:
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

Average selling price for a crate of vegetables: $8.00
Costs for seeds, labor, etc. (variable) 2.00/crate
Variable cost for running the greenhouse 1.00/crate
Annual costs for having the greenhouse (depreciation and operating costs) $250,000
Capacity within the greenhouse 20,000 crates/turn

Required:
1. What is the breakeven volume of vegetables for Frank’s plan?
2. If Frank can sell everything that he can grow, what is the maximum level of profit he can earn from this investment in a year?
3. Should Frank go ahead with this investment? How large an error could Frank have made in his calculations before you begin to feel uncomfortable about the decision?

Problem 6.
Sally’s Dry Cleaning has just bought its own shirt laundry and pressing machines. The numbers Sally used to justify the decision are:

- Projected annual revenues from laundry $500,000
- Less projected variable costs 300,000
- Projected contribution margin 200,000
- Less projected annual fixed costs 150,000
- Projected net income before tax $50,000

You’ve examined Sally’s estimates of sales volume, and you don’t feel very good about them. In fact, you are fairly certain they may be wrong. So you decide to use the techniques you have learned to see what changing the expected level of sales would do to Sally’s predictions.

Required:
1. Compute the breakeven revenue for Sally’s new laundry operation.
2. If Sally’s actual sales volume is 10% less than she plans, would she still break even? Why or why not?
3. If Sally’s actual sales volume is 20% less than planned, would she still break even?
4. If Sally’s actual sales volume is 10% more than planned, what would her profits be?
5. What would you recommend to Sally? Why?

Problem 7. (Complex Break Even! Technique explained in solution)
Well-done Frozen Foods produces two different types of frozen dinners: chicken and roast beef. Well-done sells the dinners by the case to grocery stores and wholesalers. Chicken dinners sell for $15.00 a case, while roast beef dinners sell for $18.00 a case. The variable costs for both dinners is 60% of their selling
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

price. Projected fixed costs for the company for the next year are $350,000. Well-done normally sells 50,000 cases of chicken dinners and 25,000 cases of beef dinners in a year. The fixed costs are common to both products.

**Required:**
1. Compute the sales revenue that must be earned for Well-done to break even.
2. Compute the number of cases of chicken dinners and beef dinners that Well-done must sell to break even.
3. How much money will Well-done make if it meets its sales goals?

**Problem 8.**

The following data have been condensed from Rhone Corporation's report for last year's operations [in millions of French francs (FF)]:

<table>
<thead>
<tr>
<th></th>
<th>Variable</th>
<th>Fixed</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>FF 900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing costs</td>
<td>FF 400</td>
<td>FF 180</td>
<td>FF 580</td>
</tr>
<tr>
<td>Selling and administrative expenses</td>
<td>FF 140</td>
<td>FF 60</td>
<td>FF 200</td>
</tr>
</tbody>
</table>

Top management is considering two options for next year:
1. reducing selling prices by 10 percent, which will probably result in an average increase in volume of goods sold of 30 percent, or
2. redesigning manufacturing operations, which would increase annual fixed manufacturing costs by FF 80 million and decrease variable manufacturing costs by 15 percent per unit of product.

**Required:**
1. Compute break-even in sales francs, using last year's numbers.
2. Prepare an income statement for last year using the contribution margin format.
3. Compute the effect on pretax income for each of the two options being considered for next year.
**Differential Analysis Problems:**

**Problem 9**

Sinestrics, Inc. produces and sells specialty household gadgets. The company is in the process of evaluating the potential of a new product: the left-handed can opener. The following cost structure has been estimated for each unit produced, based on an annual volume of 300,000 units. The item will require no new facilities. Production facilities for this product will be idle if this one is not produced.

<table>
<thead>
<tr>
<th>Item</th>
<th>Cost per Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct materials</td>
<td>$0.25</td>
</tr>
<tr>
<td>Direct labor</td>
<td>0.50</td>
</tr>
<tr>
<td>Factory overhead</td>
<td></td>
</tr>
<tr>
<td>Variable</td>
<td>0.30</td>
</tr>
<tr>
<td>Fixed</td>
<td>0.15</td>
</tr>
<tr>
<td>Total per unit</td>
<td>$1.20</td>
</tr>
</tbody>
</table>

Sinestrics has two alternative marketing plans for this product. Under plan A, it would be sold to specialty shops at $2.90 each. Selling costs would include a total of $0.65 per unit for commissions and average shipping costs. The promotion budget would be $200,000. Plan B would be to sell the thing at $3.69 each directly to consumers through a telemarketing plan. Variable selling costs, nearly all order-processing and shipping, would only be $0.45 per unit, but the TV promotions would total $500,000. Since the TV promotions would rely on the standard "not sold in stores" pitch, the two plans are mutually exclusive.

**Required:**

1) What is the breakeven point for the product under plan A in units? ... in dollars?
2) What is the breakeven point for the product under plan B in units? ... in dollars?
3) Suppose projected demand under plan A was 300,000 units and under plan B was 350,000. Should Sinestrics produce the left-handed can opener? If so, which plan should they use? Why?

**Problem 10**

Probable House (PH) is a major book publisher. In 1988, the company is considering whether to enter into a contract with a famous novelist, Stephen Queen, in order to acquire the rights to the writer's latest book. The contract specifies that the author would be paid an advance of $1.5 million dollars. In addition, the author is entitled to a royalty of 15 percent of the price charged to retailers.

PH plans to release the book as a hard-cover. PH would charge retailers a price of $10. Retailers would subsequently sell the book at the cover price of $20. The cost of promoting the book is expected to run to $250,000. Variable costs of production amount to $3 per book. Other variable costs (excluding
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

royalty) amount to $2 per book. Production of the book will not result in any additional fixed production costs.

Required:
1. (a) How many copies of the book must PH sell in order to break even? (b) Assuming an income tax rate of 30 percent, how many copies of the book must PH sell in order to earn income after taxes of $70,000.
2. Prepare an income statement showing the book's contribution to the company's income, using the contribution margin format. Assume sales of 200,000 copies and no income taxes.
3. Realistically, a best selling hard-cover book is unlikely to sell more than 200,000 copies. Publishers generally follow up hard-cover books with a paperback version. PH intends to sell its paperback to retailers at $3, for subsequent resale to the public at $6. Variable manufacturing costs will amount to $1.50, and other variable costs (excluding royalty) to $0.50 per book. An additional cost of $150,000 will be incurred for promotion of the paperback. Assuming that PH is able to sell 200,000 copies of the hard-cover, how many copies of the paperback must it sell to break even on the book as a whole?
4. Popular Book Club would be interested selling the book to its members. The club would be willing to pay PH $120,000 for the right to publish the book itself and sell it to club members. PH would pay the author 30 percent of this amount as royalty. No other costs would be incurred. However, PH estimates that its regular hard-cover sales would drop by 10,000 copies if it accepts the book club's offer, since many likely purchasers are book club members. Would it be profitable for PH to accept the book club's offer?

Problem 11
Doojigger, Inc. produces 10,000 units of Permazoy Diesel fuel filters each year for use in their main product, the Power Doojigger. The bill of materials for each filter is:

<table>
<thead>
<tr>
<th>Direct materials</th>
<th>$4.80</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct labor</td>
<td>7.00</td>
</tr>
<tr>
<td>Variable overhead</td>
<td>3.20</td>
</tr>
<tr>
<td>Fixed overhead</td>
<td>10.00</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>$25.00</td>
</tr>
</tbody>
</table>

An outside supplier has offered to supply Doojigger with any number of filters, per Doojigger's specifications, annually for $23.50 each. If the offer is accepted, Doojigger will be able to rent the facilities currently used for filter production to another tenant in the same industrial park for $40,000 per annum.
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

Required
At what volume of filters would Doojigger be indifferent as to whether it made or bought the filters (under cost considerations only)? What other considerations are relevant?

Problem 12
American Dumpster Corporation (ADC) manufactures and sells several kinds of trash containers. ADC is considering the discontinuance of the "Minicans" line, based on the product line income statement from last year, given below:

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>&quot;Scow&quot;</th>
<th>&quot;Drum&quot;</th>
<th>&quot;Mini&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>$2,275,000</td>
<td>750,000</td>
<td>525,000</td>
<td></td>
</tr>
<tr>
<td>less variable costs:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manufacturing</td>
<td>(900,000)</td>
<td>(325,000)</td>
<td>(175,000)</td>
<td></td>
</tr>
<tr>
<td>(400,000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Selling &amp; Admin.</td>
<td>(100,000)</td>
<td>(25,000)</td>
<td>(25,000)</td>
<td></td>
</tr>
<tr>
<td>(50,000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contribution margin</td>
<td>1,275,000</td>
<td>400,000</td>
<td>325,000</td>
<td></td>
</tr>
<tr>
<td>less fixed costs:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Direct --</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line supervision</td>
<td>(120,000)</td>
<td>(30,000)</td>
<td>(30,000)</td>
<td></td>
</tr>
<tr>
<td>(60,000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Line promotion</td>
<td>(60,000)</td>
<td>(20,000)</td>
<td>(20,000)</td>
<td></td>
</tr>
<tr>
<td>(20,000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Allocated --</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Factory overhead</td>
<td>(600,000)</td>
<td>(150,000)</td>
<td>(150,000)</td>
<td></td>
</tr>
<tr>
<td>(300,000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>General admin.</td>
<td>(400,000)</td>
<td>(100,000)</td>
<td>(100,000)</td>
<td></td>
</tr>
<tr>
<td>(200,000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income before interest and taxes</td>
<td>$ 95,000</td>
<td>100,000</td>
<td>25,000</td>
<td></td>
</tr>
<tr>
<td>(30,000)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Required:
1. What affect would dropping the "mini" line have on pre-tax profit?
2. What other considerations would be relevant to this decision?
3. What course of action would you recommend to ADC's management?
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

Suggested Solutions:

Problem 1
1. Average variable cost per unit made is the sum of all of the costs that are incurred on a unit by unit basis. This generally includes direct materials, direct labor, and any identifiable variable overhead. Since the only overhead items in this problem that can be directly expressed on a per-unit is machine time cost (all of the others are stated as $X per month, none qualify as being incurred on a unit by unit basis), this is the only variable overhead cost. Thus, the answer here is the sum of the per unit costs of DM, DL, and machine time:

<table>
<thead>
<tr>
<th>Component</th>
<th>Per Unit Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>White plastic</td>
<td>$0.30</td>
</tr>
<tr>
<td>Clear plastic</td>
<td>$0.15</td>
</tr>
<tr>
<td>Decals</td>
<td>$0.10</td>
</tr>
<tr>
<td>Total DM</td>
<td>$0.55</td>
</tr>
<tr>
<td>DL</td>
<td>$0.75</td>
</tr>
<tr>
<td>VOH (machine time)</td>
<td>$0.50</td>
</tr>
</tbody>
</table>
| Total Variable cost per unit | $1.80

2. Total fixed cost is simply the sum of the remaining (i.e., non-variable) costs -- $6,000 per month.

3. Volume: 1,000/mo 10,000/mo 50,000/mo

<table>
<thead>
<tr>
<th>Volume</th>
<th>1,000/mo</th>
<th>10,000/mo</th>
<th>50,000/mo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost calculation:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. TVC = $1.80 X Vol</td>
<td>$1,800</td>
<td>$18,000</td>
<td>$90,000</td>
</tr>
<tr>
<td>$180,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b. TFC = $6,000</td>
<td>$6,000</td>
<td>$6,000</td>
<td>$6,000</td>
</tr>
<tr>
<td>$6,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c. TC = TVC + TFC</td>
<td>$7,800</td>
<td>$24,000</td>
<td>$96,000</td>
</tr>
<tr>
<td>$186,000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d. Avg VCu = $1.80</td>
<td>$1.80</td>
<td>$1.80</td>
<td>$1.80</td>
</tr>
<tr>
<td>e. Avg FCu = TFC/Vol</td>
<td>$6.00</td>
<td>$0.60</td>
<td>$0.12</td>
</tr>
<tr>
<td>f. Avg cost per unit</td>
<td>$7.80</td>
<td>$2.40</td>
<td>$1.92</td>
</tr>
</tbody>
</table>

Problem 2
1. TC means total costs of making clown noses. The problem does not say so, but this must be per some period of time, since the fixed costs probably do not represent the ability to make an infinite number of noses.

<table>
<thead>
<tr>
<th>Volume</th>
<th>1,000</th>
<th>5,000</th>
<th>8,000</th>
<th>16,000</th>
<th>25,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC</td>
<td>$8,150</td>
<td>$8,750</td>
<td>$9,200</td>
<td>$10,400</td>
<td>$11,750</td>
</tr>
<tr>
<td>Avg C/u</td>
<td>$8.15</td>
<td>$1.75</td>
<td>$1.15</td>
<td>$0.65</td>
<td>$0.47</td>
</tr>
</tbody>
</table>
Problem 3

1. a. Direct materials is, by definition, a variable cost. Therefore, if it costs a total of $75,000 to make 50,000 orders (packets) of checks, the DM costs must be $1.50 per packet. The formula --
   \[ TDM = \$1.50 \times \text{Volume of packets} \]

   b. Direct labor is also a variable cost. Using the same kind of logic as above, the per packet cost of DL must be $10,000/50,000, or $0.20.
   \[ TDL = \$0.20 \times \text{Volume of packets} \]

   In order to do parts c and d, we need to know which overhead items are fixed and which are variable. In the "real world," many would probably be "mixed" costs, but we are told to force them into the fixed or variable categories. Using our knowledge of (1) accounting and (2) how the world works, we can conclude the following:
   * Supply use probably goes up as we do more work, so it is variable.
   * Depreciation is usually $X per year, so that is fixed.
   * Indirect labor (other than supervision) involves moving materials and goods, inspecting them, etc. The total probably goes up as more work is done. Therefore we should call it a variable cost.
   * Power runs the machines that make the goods, so as we make more, we use more power and our electric bill goes up. It meets the definition of a variable cost.
   * Supervision is the salaries of the "shop floor" managers. We pay them the same amount regardless of the number of units made, so this is a fixed cost.
   * Insurance costs are based on assets and employees and are typically charged for a whole year at a time. This puts them in the fixed cost category.
   * Rent is typically charged based on length of time the space is occupied. The landlord will not give us a reduction if we do not produce anything. Therefore, rent is a fixed cost.

   c. \[ \text{Total Variable Overhead (TVOH)} = \frac{[(5,000+2,400+500)/50,000]}{50,000} \times \text{Volume of packets} \]
   \[ = \$0.158 \times \text{Volume of packets} \]

   d. \[ TVC = \text{TDM} + \text{TDL} + \text{TVOH} = (\$1.50 + 0.20 + 0.158) \times \text{Volume of packets} \]
   \[ = \$1.858 \times \text{Volume of packets} \]

   e. \[ TFC = (3,600+2,300+700+1,500) = \$8,100 \]

   f. \[ TC = \$8,100 + (\$1.858 \times \text{Volume of packets}) \]

   g. \[ \text{Avg total cost per unit} = \frac{TC}{\text{Volume}} = (\$8,100/\text{Volume}) + 1.858 \]
Now we use the formulae above in parts 2, 3, and 4:

2. 75,000 packets (totals):
   a. TDM = $1.50 X 75,000 = $112,500
   b. TDL = $0.20 X 75,000 = $15,000
   c. TVOH = $0.158 X 75,000 = $11,850
   d. TVC = $1.858 X 75,000 = $139,350
   e. TFC = $8,100 (remember, fixed costs do not change with changes in volume!)
   f. TC = $8,100 + ($1.858 X 75,000) = $147,450

3. 75,000 packets (averages):
   a. DM/u = $1.50
   b. DL/u = $0.20
   c. VOH/u = $0.158
   d. VC/u = $1.858
   e. FC/u = $8,100/75,000 = $0.108
   f. TC/u = $1.966 (= 1.858 + .108)

4. At 100,000 packets, total costs will increase for variable costs, but stay the same for fixed costs. Average costs at this higher volume will stay the same as before for variable cost per unit, but fall for fixed cost per unit (and, by extension, total cost per unit).
   TDM = $150,000   DM/u = $1.50
   TDL = $20,000   DL/u = $0.20
   TVOH = $15,800   VOH/u = $0.158
   TVC = $185,800   VC/u = $1.858
   TFC = $8,100   FC/u = $0.081
   TC = $193,900   TC/u = $1.939

Solution to Problem 4
1. Variable cost per unit = $684,000 / 45,000 units = $15.20
   Selling price per unit = $1,710,000 / 45,000 bags = $38.00
2. Break even units = $525,000 / ($38.00 - $15.20) = 23,026 units
   Break even sales = 23,026 units x $38.00 or $525,000 / ($22.80 / $38.00) = $875,000
3. Break even units = $725,000 / ($38.00 x .70) = 27,256 units
   Break even sales = 27,256 units x $38.00 or $725,000 / .70 = $1,035,714

Solution to Problem 5
1. $250,000 / $5.00 = 50,000 crates to break even.
2. Capacity is 20,000 crates per "turn" and three turns per year. Thus, he can produce 60,000 crates all together. This number is 10,000 crates above break even. At $5.00 contribution margin per crate, that yields $50,000 profit
3. If these numbers are correct, then he should go with the new products. However, are they correct? He can have an error of up to $50,000 in annual fixed costs before he goes into a loss IF VARIABLE COSTS ARE ACCURATE.
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

This level of error in the fixed costs is unlikely if he has done his homework, so the question is how much error in his VC estimates can he tolerate? That is, how high could the VCu be before he would be operating at a loss at the expected volume of 60,000 crates? We can use the break even formula, set units sold to 60,000, and solve for CMu to give us the answer. To break even at 60,000 crates capacity, \( \text{ASSUMING FIXED COSTS ARE ACCURATE AT} \$250,000 \), he can have a CMu as low as:

\[
\frac{\$250,000}{\text{CMu}} = 60,000 \text{ crates}
\]

\[
\text{CMu} = \frac{\$250,000}{60,000} = \$4.17
\]

\( \text{SPu} - \text{CMu} = \text{VCu} \), so \( \text{VCu} = \$8.00 - 4.17 = \$3.83 \), or \$0.83 higher than currently expected.

\[\text{Solution to Problem 6}\]

1. Break even revenue = \$150,000 / .4 = \$375,000
2. Yes; Revenues = 450,000; VC = \$270,000; CM = \$180,000 - \$150,000 in fixed costs = \$30,000 in net income before taxes
3. Yes; Revenues = 400,000; VC = \$240,000; CM = \$160,000 - \$150,000 in fixed costs = \$10,000 in net income before taxes
4. Revenues = 550,000; VC = \$330,000; CM = \$220,000 - \$150,000 in fixed costs = \$70,000 in net income before taxes
5. It looks like she is fairly assured of a profit assuming her estimates of sales are "in the ballpark" and that her fixed cost/variables cost breakdown is correct.

\[\text{Solution to Problem 7}\]

1. Regardless of the type of dinner they sell, their CM is 40% of the selling price. Since breakeven is where TCM = TFC, they will breakeven when 40% of revenues is equal to the \$350,000 fixed costs:

\[
.4 \times X = \$350,000; \quad X = \$875,000 \text{ in sales to break even}
\]

2. Here we have to make an assumption. We can assume that the two kinds of dinners always sell in the same ratio (or sales mix). Thus, they sell 2 chicken dinners for every beef dinner. That means the "average" case sold is 2/3 chicken dinners and 1/3 beef dinners. IT would have a CMu of:

\[
\frac{2}{3} \times (.4 \times \$15) + \frac{1}{3} \times (.4 \times \$18) = \$4 + \$2.40 = \$6.40
\]

Thus, breakeven is \$6.40 \times X = \$350,000; so X = 54,687.5 average cases. 2/3 of that total (36,458.33) would be cases of chicken dinners and 1/3 (18,229.17) would be cases of beef dinners. Since you cannot sell partial units, we would have to say 36,459 cases of chicken dinners and 18,230 cases of beef dinners.

3. Assuming sales goals are for "normal" sales (not much of a goal!), they would sell 50,000 cases of chicken dinners (x \$6.00 CMu = \$300,000) and 25,000 cases of beef dinners (x \$7.20 CMu = \$180,000). Thus, income before taxes will be:
### Solution to Problem 8

1. Total contribution margin = 900 - (400 + 140) = FF 360  
   CM ratio = 360/900 = 0.40 (or 40 percent)  
   Required sales for break-even (in francs) = \( FC/CM \) ratio = 240/0.40 = 
   $600

2. Sales  
   Less Variable costs  
   Manufacturing  
   Selling & Adm  
   Contribution margin  
   Fixed costs  
   Manufacturing  
   Selling & Adm  
   Pretax income  

3. Reduce Selling Price  
   Redesign Operations  
   Sales  
   Variable costs  
   Manufacturing  
   Selling & Adm  
   Contribution margin  
   Fixed costs  
   Manufacturing  
   Selling & Adm  
   Pretax income  

### Solution to Problem 9

Since Sinestrics sells a variety of gadgets, we can assume that the fixed costs of manufacturing are mostly (if not all) unavoidable, committed fixed costs. Thus, we only want to know how long it takes for this product to break even above its specific marketing fixed costs. The $0.15 per unit of allocated fixed costs does not really "belong" to this product, thus we ignore it in the analysis.
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

1) \[ \text{BEPu} = \frac{\text{Marginal Fixed Costs}}{\text{unit Contribution Margin}} \]
   \[ = \frac{200,000}{(2.90-(.25+.50+.30+.65))} \]
   \[ = \frac{200,000}{1.20} = 166,666.66 \text{ units} \]
   \[ \text{BEP$} = 166,666.66 \times 2.90 = 483,333.33 \]

2) \[ \text{BEPu} = \frac{500,000}{(3.69-(.25+.50+.30+.45))} \]
   \[ = \frac{500,000}{2.19} = 228,310.50 \text{ units} \]
   \[ \text{BEP$} = 228,310.50 \times 3.69 = 842,465.74 \]

3) Both estimates are above BEP, so Sinestrics should go with manufacturing it. The highest contribution comes from the telemarketing approach, which, despite a higher BEP, earns money above breakeven faster, due to its higher CMu.

Pre-tax Profit = Total Marginal CM - Marginal Fixed Cost
A: \[ (300,000 \times 1.20) \times 200,000 = 160,000 \]
B: \[ (350,000 \times 2.19) \times 500,000 = 266,500 \text{ (or } 106,500 \text{ more pre-tax profit)} \]

Solution to Problem 10

1. Fixed cost = 1,500,000 + 250,000 = $1,750,000
   Unit variable cost = 1.50 (royalty) + 3.00 (production) + 2.00 (other) = $6.50
   Unit revenue = $10.00
   Therefore, unit contribution margin = 10 - 6.50 = $3.50
   Req'd sales for break-even = 1,750,000/3.50 = 500,000 books
   Required pretax income = 70,000/(1-0.3) = $100,000
   Req'd sales for desired profit level =
   \[ (1,750,000 + 100,000)/3.50 = 528,572 \text{ books} \]

2. Sales (200,000 X $10) $2,000,000
   Variable costs
   Royalty 300,000 (200,000 X $1.50)
   Production 600,000 (200,000 X $3.00)
   Other 400,000 (200,000 X $2.00)
   \[ \text{Contribution margin} = 1,300,000 \]
   \[ \text{Fixed costs} = 700,000 \]
   Advance 1,500,000
   Promotion 250,000
   \[ \text{Net income} = 1,750,000 \]
   \[ \text{Net income} = 1,750,000 \]
   \[ \text{Net income} = 1,750,000 \]
ANALYZING THE EFFECTS OF FIXED AND VARIABLE COSTS

3. Unit variable cost of paperback:
   1.50 (production) + 0.50 (other) + 0.45 (royalty) = $2.45
   Unit contribution margin = 3.00 - 2.45 = $0.55
   Paperback must contribute to its own fixed costs of $150,000 as well as hardcover's loss of $1,050,000, a total of $1,200,000.
   Reqd sales for break-even = 1,200,000/0.55 = 2,181,819 books

4. Contribution from Club sales = 120,000 - 36,000 (royalty of 30%) = $84,000
   Lost contribution from drop in hard-cover sales =
   10,000 X $3.50 (hard-cover's unit CM) = $35,000
   Net benefit of Club contract = 84,000 - 35,000 = $49,000

Solution to Problem 11

Cost of buying: $23.50 per filter minus $40,000 per year.
Cost of making: $15.00 per filter.
If the volume at which the two are indifferent is X, then
23.50 X - 40,000 = 15 X
so X = 40,000/8.5 = 4706 units per year.

Solution to Problem 12

1) If the product line income statement can be relied upon, we would expect all revenues and costs directly associated with the mini line to go with its elimination. The allocated costs, however, would stay. Thus, the IBIT would fall by the mini line's segment margin: 550,000 - 80,000 = 470,000. IBIT would fall to a loss of $375,000.

2) Several other issues should be considered, too. Are there alternative uses for the plant capacity? Drums, for example, have a healthier percentage contribution. Will some "fixed" costs fall? Are sales of the products associated in some way? Do distributors look for a single source for all three types of products? Would minis sell better if they were promoted more heavily? Would they sell better (or for a higher price) if their quality was higher? .. if they were packaged differently? .. if they were delivered in a more timely way? Is ADC actually willing to lay off experienced workers and supervisors?

3) Without a known alternative use for the capacity, it seems unlikely that dropping the line could be profit-maximizing. Fixed costs would actually have to be cut by more than the allocated amounts. Improvements in manufacturing and marketing are likely to yield better results.